Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

2388089987

FURTHER MATHEMATICS **9231/13**

Paper 1 Further Pure Mathematics 1 **May/June 2020**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

DC (LK) 187467/2

1 The cubic equation $7x^3 + 3x^2 + 5x + 1 = 0$ has roots α, β, γ.

(a) Find a cubic equation whose roots are $\alpha^{-1}, \beta^{-1}, \gamma^{-1}$. [3]

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(b) Find the value of $\alpha^{-2} + \beta^{-2} + \gamma^{-2}$. [2]

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(c) Find the value of $\alpha^{-3} + \beta^{-3} + \gamma^{-3}$. [2]

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2 The sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = 1$ and $u_{n+1} = 2u_n + 1$ for $n \geqslant 1$.

(a) Prove by induction that $u_n = 2^n - 1$ for all positive integers n. [5]

(b) Deduce that u_{2n} is divisible by u_n for $n \geqslant 1$. [2]

3 Let $S_n = 2^2 + 6^2 + 10^2 + \ldots + (4n-2)^2$.

(a) Use standard results from the List of Formulae (MF19) to show that $S_n = \frac{4}{3}n(4n^2 - 1)$. [4]

(b) Express $\frac{n}{S_n}$ in partial fractions and find $\sum_{n=1}^{N} \frac{n}{S_n}$ in terms of N. [4]

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(c) Deduce the value of $\sum_{n=1}^{\infty} \frac{n}{S_n}$. [1]

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4 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} k & 0 & 2 \\ 0 & -1 & -1 \\ 1 & 1 & -k \end{pmatrix},$$

where k is a real constant.

(a) Show that **A** is non-singular. [3]

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The matrices **B** and **C** are given by

$$\mathbf{B} = \begin{pmatrix} 0 & -3 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} -3 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

It is given that $\mathbf{CAB} = \begin{pmatrix} -2 & -\frac{3}{2} \\ -1 & -\frac{3}{2} \end{pmatrix}$.

(b) Find the value of k. [3]

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(c) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **CAB**. [5]

5 The curve C has polar equation $r = a\tan\theta$, where a is a positive constant and $0 \leqslant \theta \leqslant \frac{1}{4}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole. [2]

(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{4}\pi$. [4]

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(c) Show that C has Cartesian equation $y = \dfrac{x^2}{\sqrt{a^2 - x^2}}$. [3]

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(d) Using your answer to part **(b)**, deduce the exact value of $\displaystyle\int_0^{\frac{1}{2}a\sqrt{2}} \frac{x^2}{\sqrt{a^2 - x^2}}\,\mathrm{d}x$. [2]

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6 The curve C has equation $y = \frac{10 + x - 2x^2}{2x - 3}$.

(a) Find the equations of the asymptotes of C. [3]

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(b) Show that C has no turning points. [3]

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(c) Sketch C, stating the coordinates of the intersections with the axes. [3]

(d) Sketch the curve with equation $y = \left|\dfrac{10+x-2x^2}{2x-3}\right|$ and find in exact form the set of values of x for which $\left|\dfrac{10+x-2x^2}{2x-3}\right| < 4$. [6]

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7 The lines l_1 and l_2 have equations $\mathbf{r} = -5\mathbf{j} + \lambda(5\mathbf{i} + 2\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{j} + \mathbf{k})$ respectively. The plane Π contains l_1 and is parallel to l_2.

(a) Find the equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

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(b) Find the distance between l_2 and Π. [3]

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The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2.

(c) Show that P has position vector $\frac{55}{27}\mathbf{i} - 5\mathbf{j} + \frac{22}{27}\mathbf{k}$ and state a vector equation for PQ. [8]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.